VUG(3)—M(6)

2020-23

Full Marks : 80

Time : 3 hours

Answer any **four** questions in which **Q. No. 1** is compulsory

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

1. Answer *all* questions : 2×10

(*a*) The identity element in a group G is unique.

(*b*) Define abelian group with example.

(*c*) Find inverse of the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$$

(*Turn Over*)

(*d*) Find the identity element of the set S of all +ve rationals forms a group under the binary operation * defined by

$$a * b = \frac{ab}{2}.$$

(*e*) Define cosets of a subgroup H in a group G.

(*f*) If in a group every element is its own inverse then prove that the group is abelian.

(*g*) Define onto isomorphism of groups.

(*h*) Define centre set of a group.

(*i*) Prove that every cyclic group is an abelian group.

(*j*) If G_1 and G_2 are two groups, define external direct product $G_1 \times G_2$.

(*Continued*)

2. (a) Prove that the fourth root of unity is an abelian group w.r.t. multiplication. 10

(b) Prove that inverse of an element in a group is unique. 10

3. (a) If G is a multiplicative group, Prove that:

(i) $(a^{-1})^{-1} = a, \forall a \in G$

(ii) $(ab)^{-1} = b^{-1}a^{-1}, \forall a, b \in G$ 10

(b) (i) Define order of an element of a group. Find order of each element of multiplicative group $\{1, -1, i, -i\}$

(ii) Prove that order of an element a of a group is same as that of its inverse a^{-1}. 10

4. (a) If H_1 and H_2 are two subgroups of a group G then $H_1 \cap H_2$ is a subgroup of G. 10

(b) A subgroup H of a group G is a normal in G iff every left coset of H is a right coset of H in G. 10

5. (a) If G is any group and $a \in G$, then centralizer $C(a)$ is a subgroup of G. 10

(b) Prove that every subgroup of a cyclic group is cyclic. 10

6. If $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$, then prove that $A^4 = I$ the identity permutation, also prove that $\{A_1, A_2, A_3, A_4\}$ forms a cyclic group of order 4. 20

7. State and prove Cayley's theorem. 20

8. Suppose G_1 and G_2 are groups. Prove that $G_1 \times \{e_2\}$ and $\{e_1\} \times G_2$ are normal sub-groups of $G_1 \times G_2$, e_1 and e_2 are unit elements of G_1 and G_2 respectively. 20
