2020-23

Full Marks : 80

Time : 3 hours

Answer any **four** questions in which Q.No.1 is compulsory.

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer all questions : 2×10

(*i*) Define the neighborhood of a point in R.

(*ii*) What are the order properties of R ?

(*iii*) What is an isolated point in R ?

(*iv*) Define the absolute value of a real number.

(*v*) Define the limit of a sequence.

(*vi*) Write the first four terms of $\left\{\frac{n^2+1}{n}\right\}$.

(Turn Over)

(*vii*) Define oscillatory sequence with an example.

(*viii*) Define partial sum of the series.

(*ix*) Write down an example of an oscillatory series.

(*x*) Define conditional convergence of a series.

2. (*a*) Prove that every nonempty bounded below set of real numbers has an infimum in R. 10

(*b*) Prove that, $a < b \Leftrightarrow -a > -b;\ \forall a, b \in R$. 10

3. (*a*) Prove that the set of real number has Archimedean Property. 10

(*b*) The set of all rational numbers is denumerable. 10

4. (*a*) Prove that every convergent sequence converges to its least upper bound. 10

(*b*) If $a_n = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{n.(n+1)} + \ldots..$

Prove that $\{a_n\}$ is bounded monotonic increasing sequence. 10

(Continued)

5. (*a*) Prove that a convergent sequence determines its limit uniqly. 10

(*b*) If, $\{a_n\} = \{\sqrt{n+1} - \sqrt{n}\}$, find $\lim_{n\to\infty} a_n$. 10

6. Discuss the convergency of the following infinite series 20

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

7. (*a*) State and prove D' Alembert's ratio test. 10

(*b*) Find whether the series

$$\frac{x}{1!} + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + ...$$

is convergent and divergent. 10

8. (*a*) Prove that the terms of a absolutely convergent series can be rearranged without affecting its convergence. 10

(*b*) Prove by an example that rearrangement of the terms of a semi convergent series may alter its sum. 10