VUG (1)–M (1)
Calculus

2017-20

Full Marks : 80

Time : 3 hours

Answer any **four** questions, in which **Q. No. 1** is compulsory.

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer *all* questions : 2 × 10

(*a*) If $x = \cosh\left(\frac{\log_e y}{m}\right)$ prove that

$$\left(x^2-1\right)y_2 + xy_1 - m^2 y = 0.$$

(*b*) State Leibnitz's theorem.

(*c*) Write the criteria for a curve $y = f(x)$ is concave upwards and concave downwards at a point $x = c$.

(*d*) Write the value of $\int_0^{\pi/2} \cos^n x \, dx$ when n is even.

(*Turn Over*)

2. (*a*) If $y = \frac{1}{x^2+a^2}$, find y_n. 10

(*b*) If $y = a\cos(\log_e x) + b\sin(\log_e x)$. Prove that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + \left(n^2+1\right)y_n = 0.$$ 10

3. (*a*) Evaluate $\underset{x\to 0}{\mathrm{Lt}}\left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$ by using L'Hospital's rule. 10

(*b*) Find the asymptotes of the curve

$$x^3 - 2y^3 + 2x^2y - xy^2 + xy - y^2 + 1 = 0.$$ 10

4. (*a*) Evaluate $\int_0^{\pi/2} \cos^n x . \cos nx \, dx$ where n is a +ve integer. 10

(*b*) Find the length of an arc of the cycloid

$$x = a(\theta + \sin\theta) \text{ and } y = a(1-\cos\theta)$$

measured from the vertex. 10

5. (*a*) Trace the curve $3ay^2 = x(x-a)^2$ and show that the length of the loop is $\frac{4a}{\sqrt{3}}$. 10

(*Turn Over*)

(*e*) If the circle $x^2+y^2=a^2$ revolves round the x-axis, find the volume of a sphere of radius a.

(*f*) Write the equation of the tangent to the conic

$$S \equiv ax^2+2hxy+by^2+2gx+2fy$$

at the point (x_1, y_1). 2011, 13

(*g*) Find the co-ordinates of the centre of the conic $S \equiv x^2-3xy+y^2+10x-10y+21=0$.

(*h*) If $\vec{a}=2\vec{i}-3\vec{j}$, $\vec{b}=\vec{i}+\vec{j}-\vec{k}$ and $\vec{c}=3\vec{i}-\vec{k}$.

Evaluate $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$.

(*i*) Write the expression for $\vec{b}\times(\vec{c}\times\vec{a})$.

(*j*) If $\vec{r}_1=t^2\vec{i}-t\,\vec{j}+(2t+1)\vec{k}$ and

$$\vec{r}_2=(2t-3)\vec{i}+\vec{j}-t\,\vec{k}.$$

Find $\dfrac{d}{dt}(\vec{r}_1\times\vec{r}_2)$ when $t=1$.

(*b*) The cardioid $r=a(1-\cos\theta)$ revolves about the initial line. Find the surface area and the volume of the figure formed. 10

6. (*a*) If the axes be turned through 45°, find the transformed form of $3x^2+3y^2+2xy=2$. 10

(*b*) Find the polar equation of the chord of the conic $l/r=1+e\cos\theta$. Joining the two points on the conic whose vectorial angles are $\alpha-\beta$ and $\alpha+\beta$. 10

7. (*a*) Prove that $\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\right]=2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$. 10

(*b*) Show that the necessary and sufficient condition for the vector function $\vec{V}$ of the scalar variable t to have constant magnitude is 10

$$\vec{V}.\frac{d\vec{V}}{dt}=0.$$

8. (*a*) If $\vec{r}=5t^2\vec{i}+t\,\vec{j}-t^3\vec{k}$ then evaluate 10

$$\int_1^2\left\{\vec{r}\times\frac{d^2\vec{r}}{dt^2}\right\}dt.$$

(*b*) Find the tangential and normal accelerations of a moving particle on a plane curve. 10