

VPG (4)-M (15)
Integral Trans.

2016-18

Full Marks : 70

Time : 3 hours

**Answer any five questions in which
Q.No.1 is compulsory.**

The questions are of equal value.

*Candidates are required to give their answers in
their own words as far as practicable.*

1. Answer all questions :

- (a) Show that the Laplace transformation is a linear transformation.
- (b) Define a function of exponential order n .
- (c) What is Parseval's identity of Fourier transformations.
- (d) Write the Fourier sine transform of $F(x)$.
- (e) What is relation between Fourier transform and Laplace transform ?

(Turn Over)

(2)

(f) Find the Hankel transform C if $rJ_0(p, r)$ be the kernel of the transform.

(g) Find $M\{e^{-x}\}$.

2. (a) State and prove sufficient conditions for the existence of Laplace transform.

(b) Find the Laplace transform of the function $F(t) = t^n$, n being any real number greater than -1 .

3. (a) Define Stieltjes transform and establish its relation to the Laplace transform.

(b) State and prove the complex inversion formula.

4. Find the Fourier transform of any two of the following functions :

$$(i) \quad f(x) = \begin{cases} x, & |x| \leq a \\ 0, & |x| > a \end{cases}$$

$$(ii) \quad f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2-x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$$

(4)

(3)

$$(iii) \quad f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

5. (a) Prove that if $f(s)$ is the Fourier transform of

$F(x)$, then $\frac{1}{a} f\left(\frac{s}{a}\right)$ is the Fourier transform of $F(ax)$. http://www.vbuonline.com

(b) Using Parseval's identity, prove that

$$\int_0^\infty \frac{dx}{(1+x^2)^2} = \frac{\pi}{4}$$

6. (a) State and prove Mellin inversion theorem.

(b) Prove that

$$M\left[\frac{1}{x} f\left(\frac{1}{x}\right); P\right] = f^*(1-p)$$

7. (a) Prove that

$$M[\log x f(x); P] = \frac{d}{dp} f^*(p)$$

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(b) Find Mellin transform of

$$(1+x^a)^{-1}, \quad a > 0$$

8. Write and prove inverse formula for the Hankel transform.

9. Find the Hankel transform of

$$\frac{d^2 f}{dx^2} + \frac{1}{x} \frac{dt}{dx}$$

when p is the root of the equation $J_p(ap) = 0$.

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