

**VPG (4)-M (14)**  
**Dyn. Syst.**

**2016-18**

*Full Marks : 70*

*Time : 3 hours*

Answer any five questions in which  
Q.No.1 is compulsory.

*The figures in the right-hand margin indicate marks.*

*Candidates are required to give their answers in  
their own words as far as practicable.*

1. (a) Define Non-linear differential equation with  
suitable examples. 2
- (b) What do you mean by a dynamical system? 2
- (c) What is continuous and discrete time dyna-  
mical system? 2
- (d) Distinguish between Autonomous and Non-  
autonomous system. 2
- (e) State Poincare Bendixson theorem. 2
- (f) Define Poincare map. 2
- (g) Define Bifurcation and chaos. 2

( Turn Over )

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2. Solve the following system of homogenous linear  
equations with constant coefficients

$$\frac{dx}{dt} = a_1x + b_1y$$

$$\frac{dy}{dt} = a_2x + b_2y$$

where  $a_1, a_2, b_1$  &  $b_2$  are real constants, under  
the following three conditions :

The conditions are as follows :

- (i) roots are real and distinct
- (ii) roots are conjugate complex
- (iii) roots are real and equal. 14

3. Find the critical points of the following non-  
linear system

$$\frac{dx}{dt} = 8x - y^2,$$

$$\frac{dy}{dt} = -6y + 6x^2$$

Also discuss their stability. 14

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4. Discuss the stability of the non-linear differential equation <http://www.vbuonline.com>

$$\frac{d^2x}{dt^2} = -\frac{g}{l} \sin x \quad 14$$

5. Define Lyapunov function and prove that if there exists a Lyapunov function  $E(x, y)$  for the system

$$\frac{dx}{dt} = F(x, y)$$

$$\frac{dy}{dt} = G(x, y)$$

then the critical point  $(0, 0)$  is stable. If this function has the addition property that the function

$$\frac{\partial E}{\partial x} F + \frac{\partial E}{\partial y} G$$

is negative definite, then the critical point  $(0, 0)$  is asymptotically stable. 14

6. Discuss the stability of linear variational equations with variable coefficients. 14

7. Write short notes on :

(i) Phase space, limit cycles 4

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(ii) Attraction and repellers 4

(iii) Dimensions and fractals 6

8. Write a short notes on :

(a) Resonance, primary and secondary resonance. 6

(b) Manifold, stable and unstable manifold, invariant manifold. 8

9. Write short notes on :

(a) Periodicity & quasi-periodicity 4

(b) Lie Algebra & Lie groups 4

(c) Symplectic manifold & Differentiable manifold 6