VPG (1)--Math (2)

2017-19

Full Marks : 70

Time : 3 hours

Q.No.1 is compulsory and answer any **four** from **Q.Nos. 2 to 9.**

The questions are of equal value.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer *all* questions : 2 × 7

(*i*) State Bessel's inequality.

(*ii*) Define uniform convergence of a sequence of functions.

(*iii*) State Wierstrass M-test for uniform convergence.

(*iv*) Define power series.

(*v*) State Abel's theorem for power series.

(*vi*) Define Jacobian.

(*vii*) If $u = x^2 - y^2$ and $v = 2xy$, calculate $\frac{\partial(u,v)}{\partial(x,y)}$

(*Turn Over*)

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2. (*a*) State and prove Parseval's theorem for Fourier series.

(*b*) Find the Fourier cosine series which represent $f(x) = \pi - x$ in $0 < x < \pi$.

3. (*a*) Let f be a founded and g a non-decreasing function on $[a, b]$. Then prove that $f \in RS(g)$ if and only if for every $\varepsilon > 0$ there exists a partition P such that $U(p, f, g) - L(p, f, g) < \varepsilon$.

(*b*) If $f \in RS(g)$ and if $|f(x)| \le K$ on $[a,b]$, then prove that $\left|\int_a^b f dg\right| \le K[g(b) - g(a)]$.

4. (*a*) State and prove Abel's theorem for uniform convergence.

(*b*) Show that the series $\sum_{n=1}^{\infty} \frac{1}{1+n^2 x}$ is uniformly convergent in $[1, \infty[$.

5. (*a*) State and prove Cauchy's general principle of uniform convergence for a sequence of functions.

(*b*) Prove that if δ is any fixed positive number less than unity, the series $\sum \frac{x^n}{n+1}$ is uniformly convergent in $[-\delta, \delta]$

6. (*a*) State and prove Abel's theorem on power series.

   (*b*) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{\lfloor 2n}{(\lfloor n)^2} x^n$.

7. (*a*) State and prove Tauber's theorem on power series.

   (*b*) Find the power series of $\tan^{-1} x$ and deduce the sum of $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots$

8. (*a*) State and prove Young's theorem.

   (*b*) Let f be defined by

$$f(x,y)=\frac{x^2y^2}{x^2+y^2},\ x^2+y^2 \neq (0,0)$$
$$=0,\ \text{otherwise.}$$

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Show that $f_{xy}(0,0)=f_{yx}(0,0)$, although neither f_{xy} nor f_{yx} is continuous at $(0,0)$. Account for the equality.

9. State and prove the inverse function theorem.